Harvey Mudd College

11p.

FINAL REPORT

IN-26579

Subject: NASA Ames Agreement No. NCC 2-314, entitled "Free Vibration and Dynamic Response Analysis of Spinning Structures"

The proposed effort involved development of numerical procedures for efficient solution of free vibration problems of spinning structures. In particular, the following R & D task has been completed as proposed.

Task 1. An eigenproblem solution procedure, based on a Lanczos method employing complex arithmetic, has been successfully developed involving

(i) Formulation of numerical procedure
(ii) Fortran coding of the algorithm
(iii) Checking and debugging of software
(iv) Implementation of the routine in the STARS program.

Task 2. A graphics package for the E/S PS 300 as well as for the Tektronix terminals has been successfully generated and consists of the following special capabilities

(a) A dynamic response plot for the stresses and displacements as functions of time

(b) A menu driven command module enabling input of data on an interactive basis.

Task 3. The STARS analysis capability has been further improved by implementing the dynamic response analysis package that provides information on nodal deformations and element stresses as a function of time.

A number of test cases were run utilizing the currently developed algorithm implemented in the STARS program and such results indicate that the newly generated solution technique is significantly more efficient than other existing similar procedures.

The following section presents some details of the solution results.

(NASA-CR-179672) FREE VIBRATICN AND DYNAMIC RESPONSE ANALYSIS OF SPINNING STRUCTURES Final Report (Harvey Mudd Coll., Claremont, Calif.) 11 p CSCL 09B

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NUMERICAL EXAMPLES

The results of numerical analysis of two representative structures analyzed by the recently developed general purpose finite element computer program STARS[8], incorporating the current solution techniques are presented in this section. Thus, the first example relates to a square cantilever plate rotating with a uniform spin rate along an arbitrary axis. Figure 2 depicts the plate with a 6 x 6 finite element mesh, the edge along the X-axis being clamped and the structure having the following basic structural data: Young's modulus (E) = 10^7, thickness (t) = 0.1, side length (ℓ) = 10, Poisson's ration (μ) = 0.3, mass density (ρ) = 0.259×10^{-3}. A free vibration analysis of the non-spinning structure was initially performed yielding a first natural frequency (ω_1) value of 3.60 (expressed in non-dimensional parametric form defined in Table I). Subsequent analyses of the structure were performed for the cases of $\Omega_Z = 0.8\ \omega_1$ as well as for a resulting spin vector $\Omega_R = 0.8\ \omega_1$ having components $\Omega_X = \Omega_Y = \Omega_Z = 0.8\ \omega_1/\sqrt{3}$. Numerical results for these analyses are presented in Table I.

A coupled helicopter rotor-fuselage system[4] freely floating in space, as shown in Figure 3, is chosen as the next example. Associated varying stiffness and mass distributions are suitably approximated for the discrete-element modeling of the structure. Free vibration analysis was performed with the rotor spinning at 10 rad/sec ($\Omega_Y = 10$). The results are presented in Table II along with the results for the corresponding non-spinning case. Associated mode shapes are shown in Figure 4, that correspond to the non-spinning rotors.

CONCLUDING REMARKS

A generalized numerical formulation has been presented for the effective evaluation of nodal centrifugal forces in various finite elements due to any arbitrary spin rate. This, in turn, enables the derivation of element geometrical stiffness and centrifugal force matrices $\mathbf{K}_G$ and $\mathbf{K}^{\circ}$, respectively, which are vital for the free vibration analysis of spinning structures. The paper also presents the details of an improved eigenproblem solution procedure for the efficient free vibration analysis rotating structures, that effect considerable saving in solution effort when compared with the earlier procedure presented in Reference 1. A recently developed general purpose finite element computer program, STARS[8], incorporated current advances such as the techniques described herein. It has proven to be useful as an analysis tool for the solution of complex practical problems.

REFERENCES

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4. J.S. Przemieniecki, Theory of Matrix Structural Analysis, McGraw-Hill, New York, 1968.

5. W.H. Wittrick and F.W. Williams, "On the free vibration Analysis of spinning structures by using discrete or distributed mass models", Journal of Sound and Vibration, Vol. 82, 1982, pp. 1-15.

6. F.W. Williams and W.H. Wittrick, "Exact buckling and frequency calculations surveyed", ASCE Journal of Structural Engineering, Vol. 109, 1983, pp. 169, 187.

7. K.K. Gupta, Comment on "On the free vibration analysis of spinning structures by using discrete or distributed mass models", Letters to the Editor, Journal of Sound and Vibration, Vol. 86, 1983, pp. 143-144.

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FIGURE CAPTIONS

Figure 1 Triangular thin shell element spinning along an arbitrary axis

Figure 2 A cantilever square plate spinning along a specified axis

Figure 3 Coupled helicopter rotor-fuselage system

Figure 4 Helicopter mode shapes

Table I. SPINNING CENTILEVER PLATE: natural frequencies for various spin rates

Mode	Natural frequency Parameter $\gamma = \omega\ell^2\sqrt{\rho t/D}$, $D = Et^3/12(1-\mu^2)$		
	$\Omega_R = 0$	$\Omega_R = \Omega_Z = 0.8\omega_1$ $= 170.86$ rad/sec	$\Omega_R = 0.8\omega_1$ $\Omega_X = \Omega_Y = \Omega_Z = 98.65$ rad/sec
1	3.6011	10.7213	7.5612
2	8.8754	16.3519	13.4013
3	21.4989	30.9203	27.7434
4	27.5027	34.1766	31.6101
5	31.5172	40.4473	38.1578
6	54.8748	62.4531	59.8561

TABLE II. NATURAL FREQUENCIES OF A HELICOPTER

Mode Number	Natural frequencies $\omega(r/s)$ for spin rates		Mode Shape
	$\Omega_Y = 0$	$\Omega_Y = 10$	
1 thru 6	0.0000	0.0000	Rigid body
7,8	4.6374	11.7370	Rotor 1st symmetric bending
9,10	5.0407	11.8189	Rotor 1st antisymmetric bending
11,12	22.1276	22.2301	Fuselage 1st symmetric bending
13,14	27.8803	36.2412	Rotor 2nd antisymmetric bending
15,16	28.3008	37.8154	Rotor 2nd symmetric bending

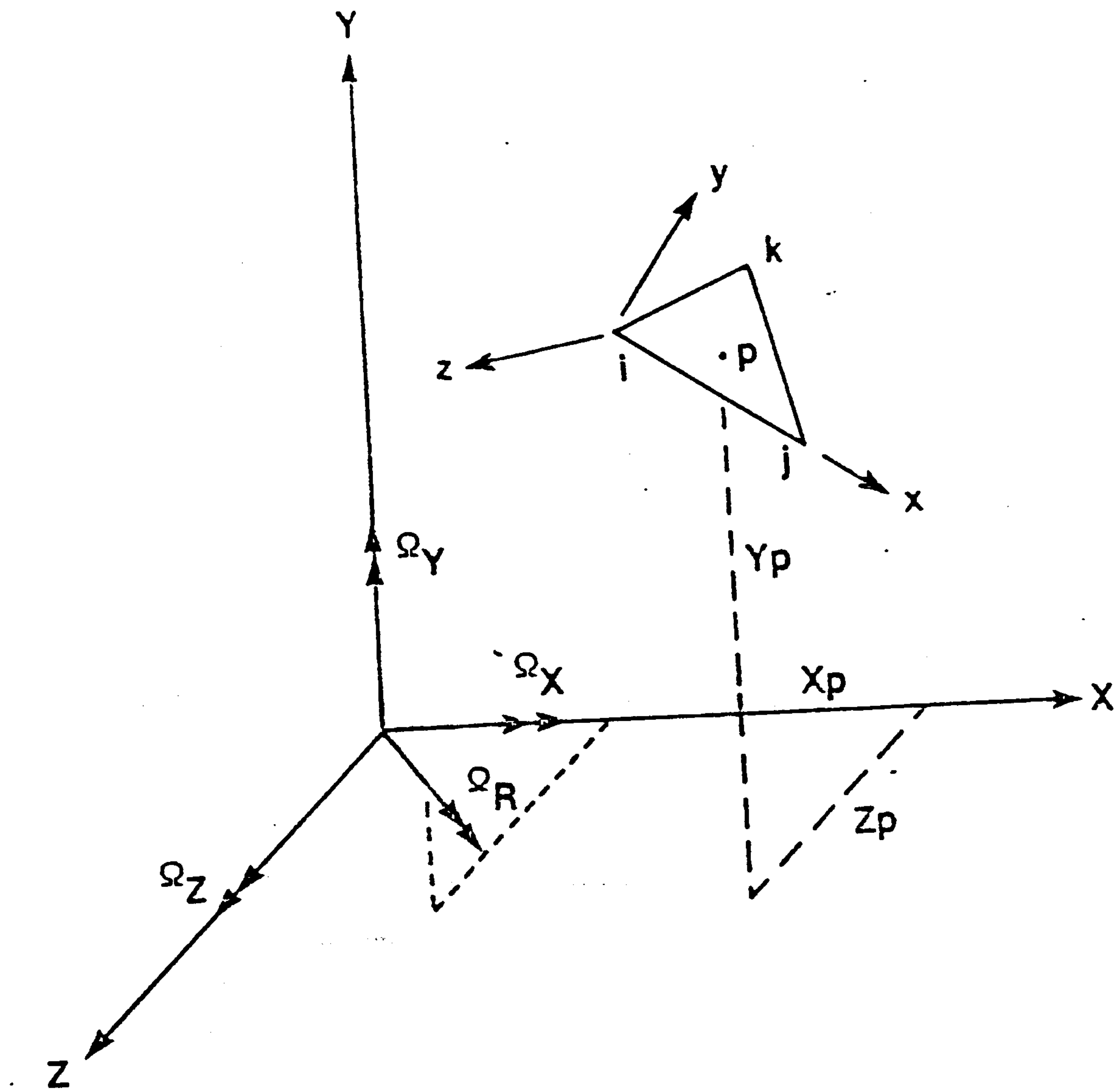

Figure 1

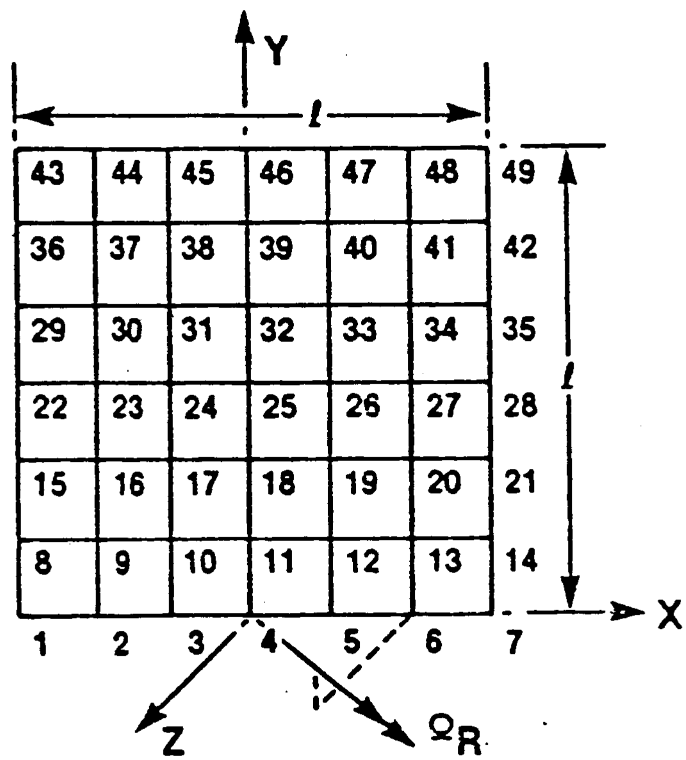
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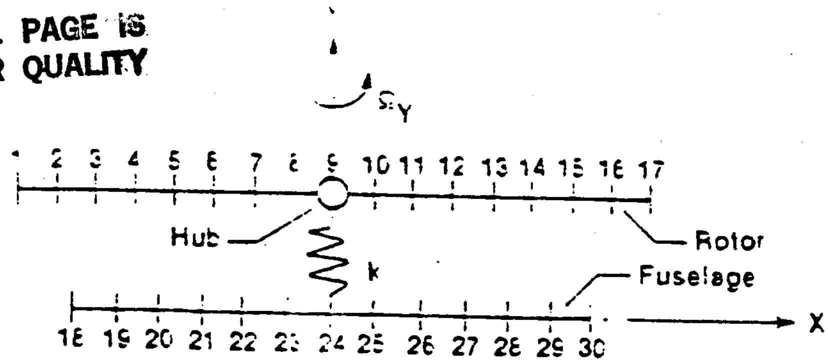
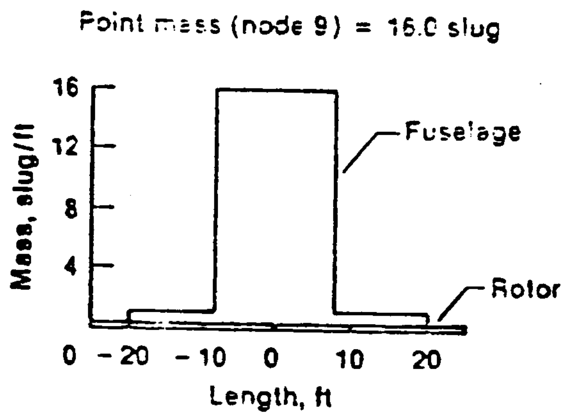
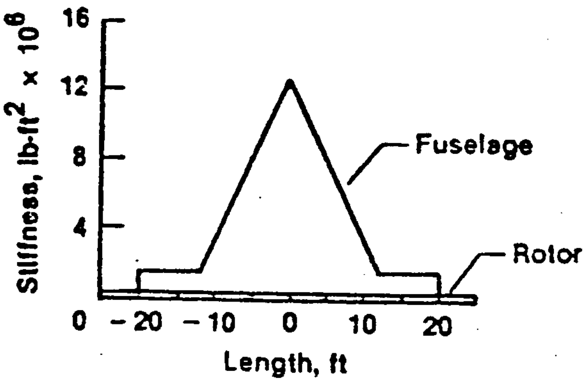
Figure 2



(a) Discrete element model



(b) Structural mass distribution



(c) Structural stiffness distribution

Figure 3

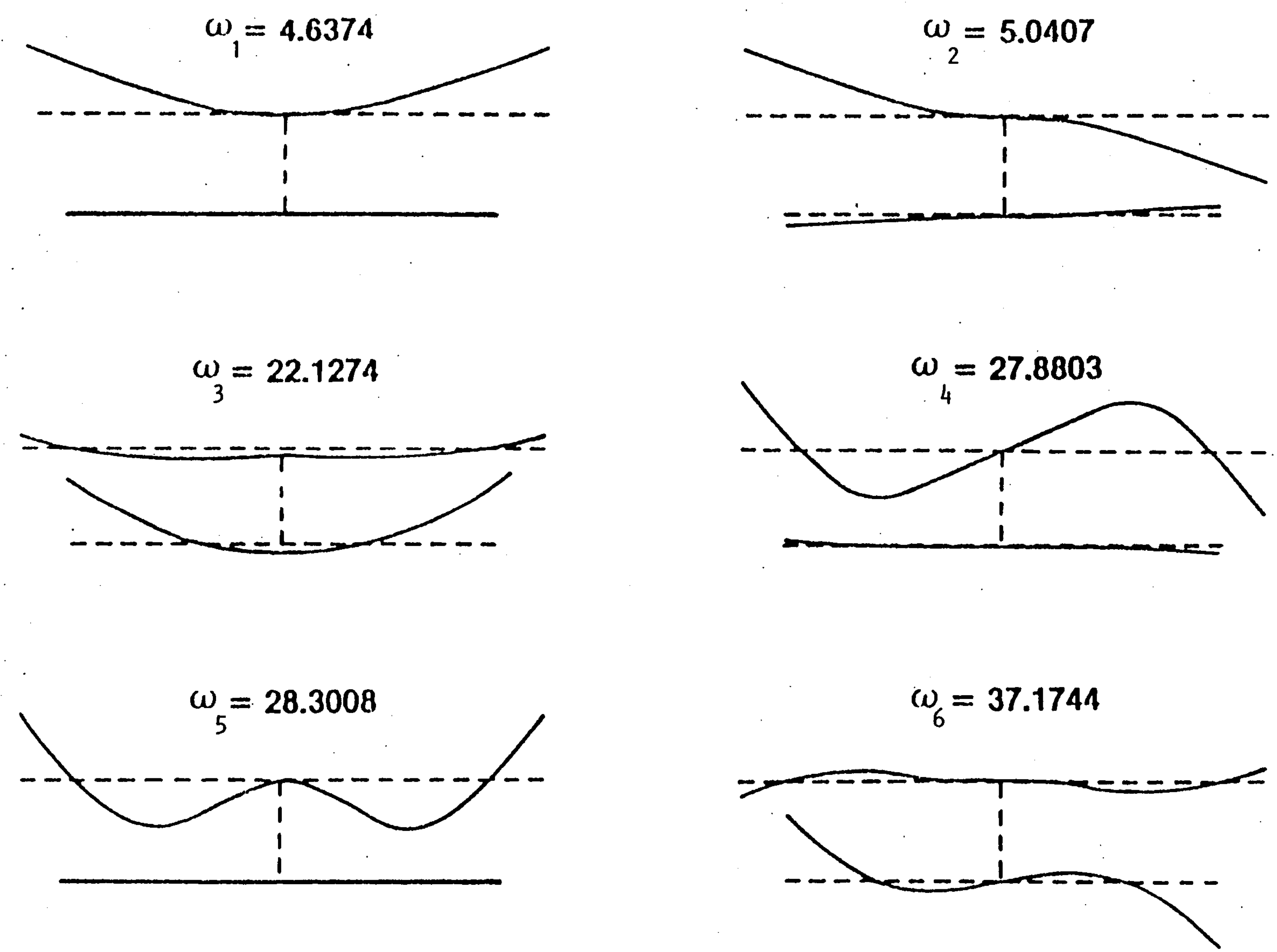

$\omega_1 = 4.6374$
$\omega_2 = 5.0407$
$\omega_3 = 22.1274$
$\omega_4 = 27.8803$
$\omega_5 = 28.3008$
$\omega_6 = 37.1744$

Figure 4